Calculus Warm Up # 9-1

Use the $\lim\limits_{x \to \pm\infty}$ to find the horizontal asymptotes of:

1. $y = e^x$

2. $y = \dfrac{7}{1 + e^{-2x}}$

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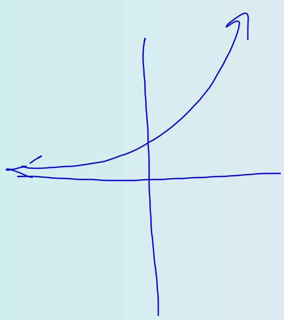
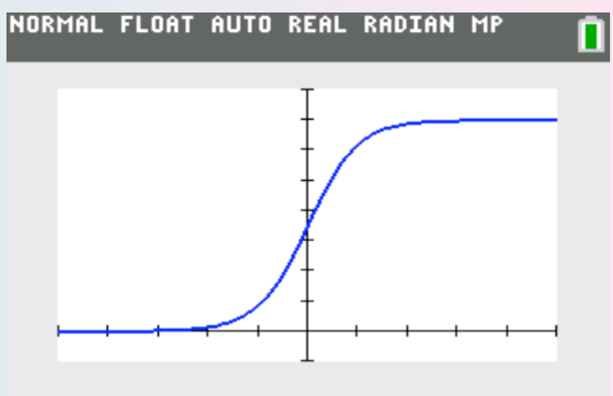
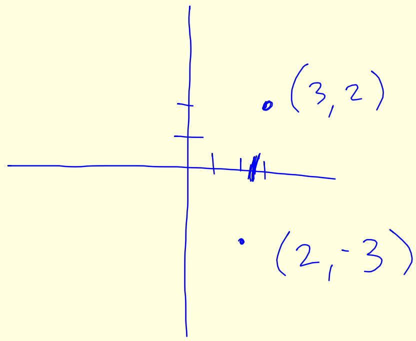


2. $y = \dfrac{7}{1 + e^{-2x}}$



Questions AP Rev WS # 3

14)



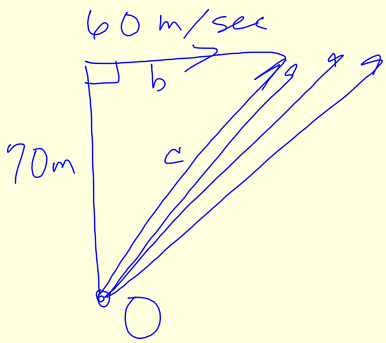
tangent

$y-2=5(x-3)$

$0-2=5x-15$

$\frac{13}{5}=\frac{\cancel{5}x}{\cancel{5}}$

81)



$70^2+b^2=c^2$

86) $f'(x)=\frac{1}{2\sqrt{x}}$

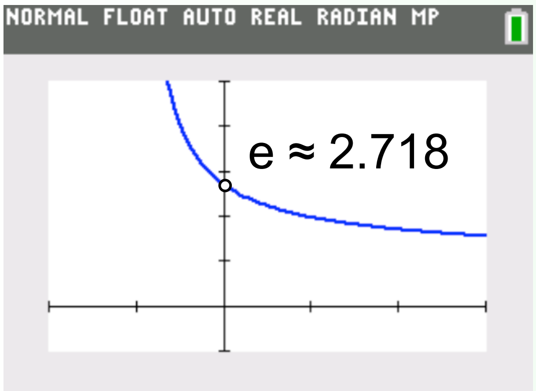
$f'(c)=2f'(1)$

From last week

The definition of e: (The natural base.)

$$\lim_{x \to 0} (1 + x)^{1/x}$$

Graph of $y = (1 + x)^{1/x}$



NORMAL FLOAT AUT
PRESS + FOR ΔTbl

X	Y_1
-.05	2.7895
-.04	2.7747
-.03	2.7602
-.02	2.746
-.01	2.732
0	ERROR
.01	2.7048
.02	2.6916
.03	2.6786
.04	2.6658
.05	2.6533

X=-.05

Use the limit definition of the derivative to find f´ :

$f(x) = e^x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^x \cdot e^{\Delta x} - e^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^x(e^{\Delta x} - 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^x(1 + \Delta x - 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^x(\cancel{\Delta x})}{\cancel{\Delta x}}$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

from def of e:

$$\lim_{\Delta x \to 0} (1 + \Delta x)^{1/\Delta x}$$

implies for small Δx

$$e^{\Delta x} = \left[(1 + \Delta x)^{1/\Delta x}\right]^{\Delta x}$$

$$e^{\Delta x} = 1 + \Delta x$$

7.2

-Differentiation of exponential functions

For $f(x) = e^x$

$f'(x) = e^x$

What about $y = e^{(3x + 2)}$?

Chain rule of course!

THEOREM 7.2 DERIVATIVE OF NATURAL EXPONENTIAL FUNCTION

Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$
2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

$y = e^{(3x + 2)}$

Derivative of the exponent expression.

Let $u = 3x + 2$

$\frac{du}{dx} = 3$

$y' = e^{(3x + 2)}\left(3\right)$

Now simplify: (just rearrange it)

$y' = 3e^{(3x + 2)}$

Evaluate:

$$\frac{d}{dx}\left[e^{-3/x}\right]$$

$$\left(e^{-3/x}\right)\left(\frac{3}{x^2}\right)$$

$$\boxed{\frac{3e^{-3/x}}{x^2}}$$

let $u = -\frac{3}{x}$

$u = -3x^{-1}$

$\frac{du}{dx} = 3x^{-2}$

$\frac{du}{dx} = \frac{3}{x^2}$

Find the relative extrema for

$f(x) = xe^{x}$

$f'(x) = xe^{x} + (1)e^{x}$

$0 = e^{x}(x+1)$

$e^{x} \neq 0 \qquad x+1=0$

$x = -1$

f' test:

$-2 \qquad -1 \qquad 0$

$+\,- \qquad\qquad +\,+$

$- \qquad\qquad +$

$\searrow \qquad \nearrow$

confirms min @ $x=-1$

$f(-1) = -1e^{-1}$

$= -\frac{1}{e}$

$$\boxed{\text{Min } \left(-1, -\frac{1}{e}\right)}$$

Plan:

find f'

set = 0

critical #'s

test

EXAMPLE 3 The normal probability density function

Show that the graph of the *normal probability density function*

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

has points of inflection when $x = \pm 1$.

let $u = -\frac{x^2}{2}$

$\frac{du}{dx} = -\frac{\cancel{2}x}{\cancel{2}} = -x$

<u>Plan</u>

find f' & f''

set = 0

test for PI

$$f'(x) = \frac{1}{\sqrt{2\pi}}\left(e^{-x^2/2}\right)(-x)$$

$$f'(x) = \left(\frac{-x}{\sqrt{2\pi}}\right)\left(e^{-x^2/2}\right)$$

$$f''(x) = \frac{-x}{\sqrt{2\pi}}\left(e^{-x^2/2}\right)(-x) + \frac{-1}{\sqrt{2\pi}}\left(e^{-x^2/2}\right)$$

$$0 = \frac{e^{-x^2/2}}{\sqrt{2\pi}}\left(x^2 - 1\right)$$

f''

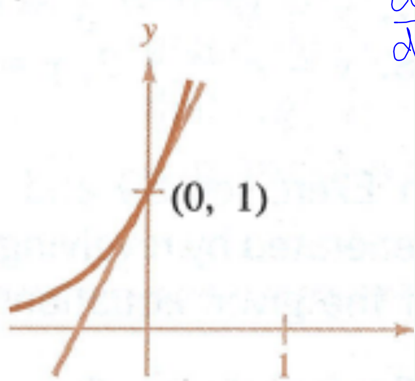
-2   -1   0   1   2

++   +−   ++

$+$ $-$ $+$

Find the slope of the tangent line at (0,1)

$$y = e^{2x}$$



let $u = 2x$

$\frac{du}{dx} = 2$

$$\boxed{y' = 2e^{2x}}$$

slope @ (0,1)

$= 2e^{2(0)}$

$= 2$

Find the derivative:

let $u = 1-x \quad \frac{du}{dx} = -1$

$u = \frac{-1}{x^2} = -x^{-2} \quad \frac{du}{dx} = 2x^{-3} = \frac{2}{x^3}$

$y = e^{1-x}$

$y' = (e^{1-x})(-1)$

$y' = -e^{1-x}$

$f(x) = e^{-1/x^2}$

$f'(x) = (e^{-1/x^2})\left(\frac{2}{x^3}\right)$

$= \frac{2e^{-1/x^2}}{x^3}$

$y = (x^2)(e^{-x})$

$u = -x$

$\frac{du}{dx} = -1$

$y' = x^2(e^{-x})(-1) + 2xe^{-x}$

$y' = xe^{-x}(-x+2)$

HW: p. 369 #1 - 27 odd

No HW Quiz tomorrow

AP Review WS # 4 due Thursday